# Impact of Magnetic Bearing Rotor Design on Satellite Nutational Stability

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**A linearized mathematical model of a three-axes stabilized symmetric satellite equipped with a magnetic bearing momentum wheel is presented. By an approximative analysis a closed form solution of the eigenvalue problem is derived which permits a quantitative assessment of bearing compliance impact on the satellite nutation. This nutational motion may become unstable due to rotor energy losses. Two various design concepts of magnetic suspension systems, i.e., passive and active magnetic bearings, are investigated for analyzing this effect. The theoretical results are verified by air bearing tests, indicating that actively controlled magnetic bearings can actually be designed to avoid the nutational instability.**

## Nomenclature

| | |
|---|---|
| $a,b$ | = general vector |
| $CM_i$ | = center of mass |
| $c_w,C,c$ | = radial bearing stiffness, related diag. matrix, normalized value |
| $d_w,D,d$ | = stator energy losses, related diag. matrix, normalized value |
| $E$ | = feedback matrix of interaxes cross coupling |
| $F$ | = electromagnetic force |
| $H_w,\boldsymbol{H}_w,H$ | = wheel angular momentum, related vector, normalized value |
| $I$ | = rotor transverse moment of inertia |
| $I_s$ | = overall transverse moment of inertia |
| $I_w$ | = rotor spin axis moment of inertia |
| $I_{1,2}$ | = inertia matrix |
| $k_w,K,k$ | = rotary damping, related diag. matrix, normalized value |
| $l_s,L,l$ | = energy dissipation from laboratory environment, related diag. matrix, normalized value |
| $Q_i$ | = vector of external excitation |
| $q_i$ | = vector of internal interaction |
| $S$ | = direction cosine matrix |
| $t$ | = time |
| $T$ | = kinetic energy |
| $T_i,\bar{T}$ | = external torques, complex representation |
| $U$ | = unit matrix |
| $v$ | = vector of linear velocity |
| $\alpha$ | = ratio of transverse rotor moment of inertia with respect to overall moment of inertia |
| $\Gamma_i,\bar{\Gamma}$ | = state vector, complex representation |
| $\delta,\gamma$ | = relative angles of rotor tilt motion |
| $\delta_i,\omega_i$ | = real part, imaginary part of $\lambda$ |
| $\bar{\delta}_i,\bar{\omega}_i$ | = real part, imaginary part of $\lambda$ of a free rotating body |
| $\epsilon$ | = feedback gain of interaxes cross coupling |
| $\lambda$ | = complex eigenvalue |
| $\rho$ | = location vector to inertial reference |
| $\bar{\phi}$ | = complex representation of overall state |
| $\varphi,\theta,\psi$ | = attitude angles of the overall system |
| $\Omega$ | = wheel rotation speed |
| $\omega_0,\boldsymbol{\omega}_0$ | = orbital angular velocity, related vector |
| $\boldsymbol{\Omega},\boldsymbol{\omega}$ | = vector of relative, absolute angular velocity |

*Subscripts*

| | |
|---|---|
| $e$ | = interaxes cross coupling |
| $n,p$ | = rotor nutation, rotor precession |
| $o$ | = fixed base value |
| $r$ | = rotary damping |
| $s$ | = overall system |
| $w$ | = wheel |
| $x,y$ | = overall transverse axes |

*Special Notation*

| | |
|---|---|
| $(\cdot)$ | = derivation with respect to time |
| $(T)$ | = matrix transposed |
| $(\sim)$ | = matrix notation of vector cross product: |

$$a=[a_1,a_2,a_3]^T\rightarrow\tilde{a}=\begin{bmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{bmatrix}$$

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## Introduction

To meet the accuracy demands of modern telecommunications satellites, much effort has been spent in Europe to increase the momentum storage capacity of common flywheels by the use of new bearing technologies. Since contactless magnetic bearings require no lubrication and since their lifetime is not limited by mechanical abrasion, they are of special significance for use in such spacecraft mechanisms as reaction wheels and high-speed rotors, which require high reliability and long-time operation. Magnetic bearing momentum wheels belong to the class of flywheels with compliant bearings. That means that contrary to flywheels of the ball bearing type, there exists a small residual motion of the rotor relative to the satellite fixed bearing elements. This motion has a significant impact on the dynamic behavior of the satellite-wheel system, e.g., due to bearing compliance the European Orbital Test Satellite OTS2 revealed an unstable nutation while stabilization was provided by a compliant grease bearing wheel.[7]

The destabilizing impact of bearing compliance during high-speed rotation was first encountered in the design of turbo rotors with elastic shafts.[1] Here a dynamic instability was identified due to nonconservative forces developed between two mating surfaces when undergoing relative sliding, referred to as rotating or rotary damping. In satellite application, the problem of stability in the presence of nonconservative agencies on rotating elements was first investigated in deriving attitude stability criteria for dual spin spacecraft.[2-4] The results of these investigations have been stability diagrams, depending on the relative angular velocity between the main body and the rotor. Moreover, there has been an essential dependence upon the ratio of the moments of inertia along the nominal spin axis with respect to the overall transverse moment of inertia. This was due to the fact that the damping forces have been assumed to act on the rotating part and/or the despun portion of the satellite only, while energy transfer by the connecting mechanism was excluded.

To evaluate the impact of this mechanism, the dynamic behavior of a despun satellite equipped with a compliant bearing rotor was investigated by Mingori.[5] Analyzing the linearized equations of motion he determined the stability boundaries of the system via the direct method of Liapunov. He could demonstrate that even a small amount of damping acting on the rotating parts of the bearings can induce overall nutational instability. Moreover, he revealed that there is no dependence of satellite nutational stability upon the overall moment of inertia along the nominal spin axis of the rotor. Therefore, rotor bearing compliance can lead to instability of the satellite nutation independent of an oblate or prolate overall configuration. Using energy sink analysis Tonkin[6] could demonstrate that this result is also achieved by an evaluation of the nonlinear equations of motion.

The investigation of the dynamics using the methods of Mingori and Tonkin yields the stability regions of the system. The decision, however, whether the nutational instability is tolerable in practice or not, the definition of design goals, etc., need a more quantitative analysis now reduced to the special device under consideration. The linearized mathematical model of the despun satellite equipped with a compliant magnetic bearing momentum wheel with high-speed rotation is presented. By an approximate eigenvalue analysis, it permits a quantitative assessment of bearing compliance impact on the satellite nutation. In case of passive bearing technology, it confirms the results derived by Mingori and Tonkin. But, moreover, it demonstrates that the stability criteria are significantly changed utilizing active magnetic bearings.

## Dynamic Model

Considering the rotations of a general two-body system (refer to Appendix) the linearized formulation of the inertial and gyroscopic loads acting between a three-axes stabilized satellite and a compliant bearing flywheel can be derived in a general form. A proper incorporation of the bearing characteristics leads to a simplified model. It represents the impact of the compliant bearing configuration by design parameters, which in general, are a function of the rotation speed.

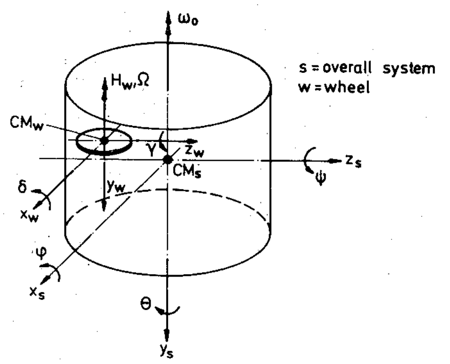


**Fig. 1 Satellite-wheel coordinate frames.**

#### Equations of Motion

Following the derivation procedure outlined in Appendix, the linearized matrix-vector differential equations of the six rotational degrees of freedom of a general gyroscopic two-body system are given by

$$I_s\ddot{\Gamma}_s + \tilde{H}_w^T\dot{\Gamma}_s + \tilde{H}_w^T\tilde{\omega}_0\Gamma_s + I_w\ddot{\Gamma}_w + \tilde{H}_w^T\dot{\Gamma}_w + \tilde{\omega}_0^T\tilde{H}_w\Gamma_w = Q_s + \tilde{H}_w\omega_0 \tag{1}$$

$$I_w\ddot{\Gamma}_s + \tilde{H}_w^T\dot{\Gamma}_s + \tilde{H}_w^T\tilde{\omega}_0\Gamma_s + I_w\ddot{\Gamma}_w + \tilde{H}_w^T\dot{\Gamma}_w + \tilde{\omega}_0^T\tilde{H}_w\Gamma_w = Q_w - q_w - \dot{H}_w + \tilde{H}_w\omega_0 \tag{2}$$

Equation (1) describes the dynamics of the overall system. The first three terms are the well-known dynamic and gyroscopic loads usually considered in investigating satellites stabilized by ball bearing flywheels. The remaining left side terms represent the coupling of overall attitude behavior with the compliant rotor motion governed by Eq. (2).

Presuming that the wheel angular momentum $|H_w|$ is maintained near a nominal value by a control loop, i.e., $|\dot{H}_w \cdot \Delta t| / |H_w| \ll 1$, the matrix of gyroscopic coupling $\tilde{H}_w$ is constant. Then the left sides of Eq. (1) and (2) comprise only the time varying vectors of state variables $\Gamma$ and their time derivatives, respectively, with constant coefficients. On the right side, the $Q$ vectors comprise the external excitations acting on the overall system and the rotor. To perform a stability analysis, those external excitations have to be regarded which are dependent on the system state or its time derivative, thereby, modifying the eigenvalues significantly. Of most concern is modeling of the nonconservative and restoring agencies acting between satellite and wheel. They are introduced in Eq. (2) by the vector $q_w$. This can be accomplished either implicitly by using additional state dependent terms in a simplified model, as it is done here, or explicitly by adding more differential equations, describing the controller state of active bearings.[10]

The resolution of Eqs. (1) and (2) to a tractable component form depends on the actual kinematics. From Fig. 1 the angular motion of the rotor about its transverse axes is given by

$$\Gamma_w = [\delta, 0, \gamma]^T \tag{3}$$

while the overall attitude behavior can be written

$$\Gamma_s = [\varphi, \theta, \psi]^T \tag{4}$$

Considering geostationary satellites, the vector of wheel angular momentum $H_w$ and the orbit angular velocity $\omega_0$ *are directed along the negative* $y$-axis, yielding

$$H_w = [0, -H_w, 0]^T, \qquad H_w = I_w\Omega \tag{5}$$

and

$$\omega_0 = [0, -\omega_0, 0]^T$$

while the vector cross product $\tilde{H}_w\omega_0 = 0$.

#### *External Excitation*

Neglecting the interaction of the overall system and the rotor with the gravitational and geomagnetic field, in space there is usually no external excitation changing the eigenvalues of the free motion. But during air bearing tests on Earth the overall system is subjected to dissipative torques resulting from the environmental atmosphere and the air bearing itself. As a first-order approximation, therefore, the $Q$ vectors can be written

$$Q_s = -L\dot{\Gamma}_s, \qquad Q_w = 0 \tag{6}$$

The matrix $L$ of the environmental damping coefficients is not time dependent, assuming small values of $\Gamma_s$ and $\dot{\Gamma}_s$. Normally, magnetic bearing momentum wheels on the ground have to be operated in a vacuum chamber and no direct external rotor excitation $Q_w$ exists.

*Internal Interaction*

Due to the assumptions made above, the relative rotor motion is determined only by the physical properties of the magnetic bearings. They can be described by

$$q_w = C\Gamma_w + D\dot{\Gamma}_w + q_r \tag{7}$$

In Eq. (7), $C\Gamma_w$ is the restoring, conservative torque producing the radial bearing stiffness. Contrary $D\dot{\Gamma}_w$ describes dissipative torques exerted on the rotor due to eddy current losses in the satellite fixed parts of the wheel. Moreover, eddy current losses are also present in the rotating parts and are comprised by $q_r$. Transforming the components of the relative state vector $\Gamma_w$ into a rotor fixed coordinate frame, the corresponding torques are proportional to the relative angular velocity ${}^w\dot{\Gamma}$ of the satellite, as it is seen from the rotor. Taking into account, that the rotor angular velocity $\Omega$ is not small, $q_r$ is given by

$$q_r = K^w\dot{\Gamma}, \qquad {}^w\dot{\Gamma} = \dot{\Gamma}_w - \tilde{\Omega}\Gamma_w \tag{8}$$

The second part of Eq. (8) represents Euler's rule of time differentiation with respect to rotating frames in matrix-vector notation. This formulation of rotary damping was first encountered in Ref. 1 and verified for magnetic bearings in Ref. 8. Often the product $K\tilde{\Omega}$ is called the "transverse stiffness," although the corresponding torques are not conservative.

In the dynamics of satellites equipped with compliant bearing flywheels, stability of the overall nutation is the most stringent problem. The following investigations are, therefore, restricted to the transverse motion of the overall system and the rotor itself. Inserting Eq. (3)-(8) into Eqs. (1) and (2), and supposing overall symmetry, the equations of motion in component form are given by

$$I_s\ddot{\varphi} + H_w\dot{\psi} + l_s\dot{\varphi} + \omega_0 H_w\varphi + I\ddot{\delta} + H_w\dot{\gamma} + \omega_0 H_w\delta = T_x$$
$$I_s\ddot{\psi} - H_w\dot{\varphi} + l_s\dot{\psi} + \omega_0 H_w\psi + I\ddot{\gamma} - H_w\dot{\delta} + \omega_0 H_w\gamma = T_z \tag{9}$$

$$I\ddot{\varphi} + H_w\dot{\psi} + \omega_0 H_w\varphi + I\ddot{\delta} + H_w\dot{\gamma} + (d_w + k_w)\dot{\delta}$$
$$+ (c_w + \omega_0 H_w)\delta + k_w\Omega\gamma = 0$$
$$I\ddot{\psi} - H_w\dot{\varphi} + \omega_0 H_w\psi + I\ddot{\gamma} - H_w\dot{\delta} + (d_w + k_w)\dot{\gamma}$$
$$+ (c_w + \omega_0 H_w)\gamma - k_w\Omega\delta = 0 \tag{10}$$

while the pitch equation of motion is decoupled and of no significance as long as $\dot{H}_w = 0$. Equation (9) is related to the roll and yaw motion of the overall system while Eq. (10) describes the corresponding relative motion of the rotor.

**Approximative Eigenvalue Analysis**

The complete set of differential Eqs. (9) and (10) is of the eighth order due to the orbit angular velocity coupling. Assuming $\omega_0$ to be very small, $\omega_0 \approx 0$, it is possible to achieve a considerable reduction of the system order. Introduction of the complex variables

$$\bar{\phi} = \varphi + j\psi, \quad \bar{\Gamma} = \delta + j\gamma, \quad \bar{T} = T_x + jT_z, \quad j = \sqrt{-1} \tag{11}$$

permits the transformation of Eqs. (9) and (10) to a third-order set of normalized equations

$$\ddot{\bar{\phi}} - j\alpha H\dot{\bar{\phi}} + \alpha l\dot{\bar{\phi}} + \alpha\ddot{\bar{\Gamma}} - j\alpha H\dot{\bar{\Gamma}} = \bar{T}/I_s$$
$$\ddot{\bar{\phi}} - jH\dot{\bar{\phi}} + \ddot{\bar{\Gamma}} - jH\dot{\bar{\Gamma}} + d\dot{\bar{\Gamma}} + c\bar{\Gamma} - jk\bar{\Gamma} = 0 \tag{12}$$

where the following definitions have been used

$$\alpha = I/I_s, \quad H = H_w/I, \quad l = l_s/I,$$
$$d = (d_w + k_w)/I, \quad c = c_w/I, \; k = k_w\Omega/I. \tag{13}$$

Considering momentum wheels the expensive magnetic bearing technology is usually restricted to systems, which demand large momentum storage capacity, together with a long lifetime. The corresponding high rotation speeds of up to 15,000 rpm limit the diameter of the rotor disk due to the centrifugal forces acting in the transverse plane. Therefore, the ratio of rotor and overall system transverse moments of inertia can be assumed $\alpha \ll 1$. The complex characteristic equation of the system of differential Eq. (12) can be written

$$\lambda^3 + [d - jH]\lambda^2 + [c - jk]\lambda + \alpha\{l\lambda^2 + [ld - jH(d+l)]\lambda$$
$$+ [(lc - Hk) - j(lk + Hc)]\} = 0 \tag{14}$$

An analytical solution of Eq. (14) in a closed form seems to be difficult. But with some simplifications due to the physical characteristics of the system, good approximations to the exact eigenvalues can be found.

*Approximation with $\alpha = 0$ (Fixed Base)*

With $\alpha = 0$ meaning no motion of the satellite about the transverse axes (= fixed base), Eq. (14) is reduced to

$$\lambda^2 + [d - jH]\lambda + [c - jk] = 0 \tag{15}$$

with the solution

$$\lambda_{1,2} = \frac{1}{2}(-d + jH)(1 \pm \sqrt{z}), \quad z = 1 - 4\frac{c - jk}{(d - jH)^2} \tag{16}$$

Assuming that $|4(c-jk)/(d-jH)^2| < 1$, which is usually true for a large angular momentum and small values of the coupling factor $k$, power series expansion of $\sqrt{z}$ yields

$$\sqrt{z} = \pm \sum_0^\infty \binom{1/2}{n}(z-1)^n = \pm\{1 + \tfrac{1}{2}(z-1) \mp \ldots\},$$
$$|z - 1| < 1 \tag{17}$$

Denoting the rotor nutation by $\lambda_n = \delta_n + j\omega_n$ and its precession by $\lambda_p = \delta_p + j\omega_p$ and inserting Eq. (17) into Eq. (16) yields

$$\delta_{p,0} = -\frac{cd + kH}{H^2 + d^2}, \quad \omega_{p,0} = -\frac{cH - dk}{H^2 + d^2},$$
$$\delta_{n,0} = -d - \delta_{p,0}, \quad \omega_{n,0} = H - \omega_{p,0} \tag{18}$$

where the subscript $0$ denotes fixed base eigenvalues. Due to the complex representation the sign of $\omega_p$ is negative, indicating the rotor precession to be retrograde to the rotor nutation. Moreover, Eq. (18) reveals that

$$\lambda_{n,0} = \bar{\lambda}_{n,0} - \lambda_p, \quad \bar{\lambda}_{n,0} = -d + jH \tag{19}$$

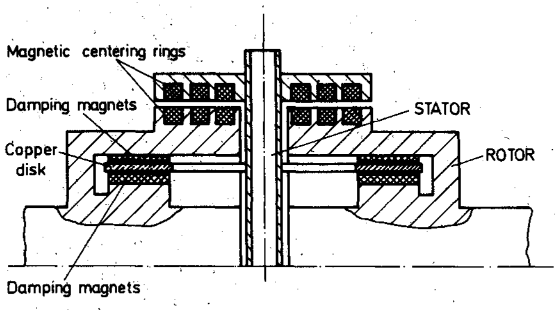


**Fig. 2 Passive magnetic bearing principle, RCPM 50.**

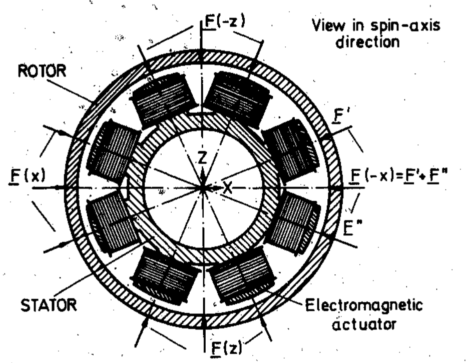


**Fig. 3 Active magnetic bearing principle, MDR 100-1.**

Here, $\lambda_{n,0}$ denotes the well-known eigenvalue of a free rotating body without any precession (c = 0). This relationship is now used to compute the eigenvalues of the overall system ($\alpha \neq 0$).

*Approximation with $\alpha \neq 0$, $\alpha \ll 1$ (Moving Base)*

Nutation frequency and damping of a satellite equipped with a ball bearing flywheel are well known. Using the notation of Eq. (13) they are given by

$$\bar{\lambda}_s = \bar{\delta}_s + j\bar{\omega}_s, \quad \bar{\delta}_s = -\alpha l, \quad \bar{\omega}_s = \alpha H \tag{20}$$

Note that $l$ was introduced to include ground testing on air bearing platforms, whereas in space $l = 0$. Starting with Eqs. (18) and (20), and using Eq. (19), it is assumed that with $\alpha \neq 0$, the overall system and relative wheel motion eigenvalues can be written

$$\lambda_n = \lambda_{n,0} - (\lambda_{p,0} + \Delta\lambda_p), \quad \lambda_p = \lambda_{p,0} + \Delta\lambda_p,$$
$$\lambda_s = \bar{\lambda}_s + \Delta\lambda_s \tag{21}$$

In Eq. (21), $\Delta\lambda_p$, $\Delta\lambda_s$ are additional terms which describe the disturbance of the free satellite nutation by the relative rotor motion and vice versa. Factorization of the characteristic equation using Eq. (21) and comparing its coefficients with Eq. (14) results in a system of four coupled nonlinear algebraic equations to determine $\Delta\delta_p$, $\Delta\omega_p$ and $\Delta\delta_s$, $\Delta\omega_s$. Taking into account some simplifications due to the expected order of magnitude of the bearing design parameters, it yields the overall nutation eigenvalues.

$$\delta_s = \alpha^2 H^3 \frac{k}{[c + 2\alpha H^2]^2} - \alpha l$$

$$\omega_s = \alpha H \left(1 - \alpha \frac{H^2}{(c + 2\alpha H^2)}\right) \tag{22}$$

From Eq. (22) it can be concluded that the nutation frequency of satellites equipped with compliant magnetic bearing wheels is always below the corresponding frequency in case of stabilization by ball bearing wheels. But, more importantly, rotary damping of passive bearing systems in space ($l = 0$) leads to $\delta_s > 0$, i.e., instability of the satellite nutation, since $k > 0$ always.[8] The instability is caused by a phase shift between the restoring torques of radial bearing stiffness and nonconservative, circulatory torques caused by rotor energy losses. The circulatory torques are represented by the skew symmetric terms $k_w$ in Eqs. (9) and (10), while the effect of energy dissipation $d_w$ in the satellite bearing elements on $\delta_s$ is negligibly small. Since the sign of $k$ cannot be changed by passive means due to the physics of the system, all despun satellites stabilized by passive complaint bearing wheels reveal this instability.

Using active bearing technology the restoring torques are commanded by a controller. Therefore, it should be possible to adjust a phase shift between conservative and nonconservative bearing torques, changing the sign of the cross coupling $k$ and thereby avoiding the instability.

The results given in Eq. (22) have been derived assuming a complete symmetric overall inertia configuration. Supposing that at least symmetry is true for the rotor mass distribution, there is no distinguished position of the satellite fixed coordinate system relative to the rotor fixed frame as long as both transverse planes are nearly parallel ($I_{xy} = I_{yz} = 0$). Therefore, the stability analysis can be performed using a satellite fixed frame with $I_{xz} = 0$ without loss of generality. Due to the symmetry of the rotor construction the results concerning stability are also valid with $I_{xz} \neq 0$. The eigenvalue solutions given above then yield qualitative results.

A similar consideration holds for $I_x \neq I_z$. This unsymmetry in the satellite mass distribution changes the shape of the overall nutation and quantitatively the degree of instability, but not the functional dependence on the wheel parameters. Indeed, on performing air bearing tests, it is impossible to balance the platform to a completely symmetric mass distribution. However, keeping $I_{xz}$ small and using an approximate overall moment of inertia $I_s = \sqrt{I_x I_z}$ yields rather good results in evaluating the measurement data as demonstrated in the next section.

## Experimental Verification

The theory above has been verified by air bearing testing of two actual wheel configurations using different magnetic bearing technology.

### Magnetic Bearing Principles

The RCPM 50, developed by SNIAS, France, is a single degree of freedom active magnetic bearing wheel, which uses permanent magnets to control the transverse rotor translation and the rotor tilt motion. The bearing principle is depicted in Fig. 2.

Radial magnetic centering rings made of ferrite, special duty alloys, and rare earth magnets create a radial adjusting force when the rotor centering ring axis is not aligned with the stator centering ring axis. The axial attraction of both the stator and rotor magnetic rings generates an axial instability along the nominal spin axis. The stability required is achieved by a one-axis electronic servo loop, which minimizes energy consumption. The damping system consists of entirely passive eddy current dampers.

In the nominal position the rotor centering axis coincides with the stator centering ring axis (i.e., the symmetry axis of the magnetic field of the bearings). Each point of the rotor disk sees a uniform magnetic field, which does not vary with time and no eddy currents are excited. During a deflection about the transverse axes, however, these two axes do not coincide any longer. Now each radius of the rotor disk passes through a stator fixed magnetic field which is nonuniform and therefore time varying, as seen from a rotor fixed frame. This develops eddy currents in the disk which, according to Lenz's law, result in nonconservative circulatory forces and torques as given by Eq. (8). The coefficient of rotary damping $k_w$ can be computed approximately from the material parameters of the bearings.[8]

Contrarily, the five-degree-of-freedom-active magnetic bearing wheel MDR 100-1,† developed by TELDIX, Germany, and SEP, France, performs the control by two electromagnetic systems. Two radial bearing segments, one above and the other below the rotor center of mass, produce the forces and torques of transverse control. One of these segments is depicted in Fig. 3.

An additional set of axial bearings stabilizes the rotor spin axis direction. The design of the bearing control system is rather complex and given in more detail in Ref. 10. Inductive sensors provide a measurement of the gap width between the rotor and the electromagnets, which is resolved to a translation and tilt angle signal by sum and difference networks. Separate translation and tilt controllers are used according to the different dynamics of these degrees of freedom. Finally the nonlinear (quadratic) characteristic of the actuator coils must be compensated by linearization networks with an approximate square root characteristic.

When rotor and stator are in nominal position with zero deflection angle there is no magnetic field at all, since no electromagnetic coil is activated. Also, there are no eddy current losses. During a deflection about a transverse axis, a single stator fixed restoring magnetic torque is generated. It is produced by a corresponding pair of coils in the transverse axis perpendicular to the axis of deflection while all other coils remain inactivated. The stator fixed magnetic field is nonsymmetric and, again, each point of the rotor disk passes through a nonuniform time varying field which excites eddy currents. This physical description of active bearing rotary damping comprises the basic effects. The actual behavior, however, is much more complex, since there is a strong feedback of the rotor on the magnetic fields of the coils.

**Test Results, RCPM 50**

The eigenvalue $\delta_s$, describing the damping of the overall system nutation in not normalized representation, is given from Eq. (22) by

$$\delta_s = \frac{k_w \Omega^4 I_w^3}{I_s^2 (c_w + 2H_w^2 / I_s)^2} - \frac{l_s}{I_s} \tag{23}$$

Figure 4 shows the corresponding results of the air bearing tests. It demonstrates that actual air bearing platform nutation in the laboratory on Earth is stable, but that damping decreases with increasing wheel speed. A least square curve fit to the measurement data using Eq. (23), augmented by pendulum tests of the air bearing platform with zero wheel speed to identify $l_s$, reveals that stability is provided by the laboratory environment, but not the bearing torques. The wheel data extracted from the measurement data are given in Table 1.

Therefore, it must be pointed out that in space, with $l_s = 0$, satellites equipped with passive magnetic bearing momentum wheels reveal the following characteristics:

1) The overall nutation is unstable, $\delta_s > 0$, and this problem becomes more significant with increasing wheel speed according to $\delta_s \sim \Omega^4$.

2) Instability is not a function of ratio of moments of inertia but depends only on the overall transverse moment of inertia, due to $\delta_s \sim 1/I_s^2$. This means the problem of instability is less significant for large satellites.

†Testing of the MDR 100-1 has been performed using the property of the International Telecommunications Satellite Organization (INTELSAT). Any views expressed are not necessarily those of INTELSAT.

In passive magnetic bearing design a large radial bearing stiffness is necessary, while rotary damping must be kept as small as possible to reduce $\delta_s$. The open loop instability has to be compensated by the satellite attitude control system, which leads to an increased power consumption. The decision has to be made in any actual case as to whether this is tolerable or not.

The RCPM 50, for example, was discussed to be used with the attitude control system of the French/German direct broadcasting satellite TV-SAT with a projected operation time of about seven years. Defining the time constant of the open loop instability analogous to the time constant of damped systems, $\tau = 1/\delta_s$, it was decided that the additional power consumption is acceptable if $\tau \geq 10$ days. With the bearing parameters given in Table 1, the RCPM 50 is feasible with an overall transverse moment of inertia $I_s \geq 1000$ kg$^2$.

**Test Results, MDR 100-1**

As long as the magnetic bearing controller poles and zeros are far enough from the characteristic eigenvalues of the dynamic system it can be demonstrated that Eq. (22) is also valid with active bearings. As a first order approximation it is sufficient to take the parameters of radial bearing stiffness $c = c(\Omega)$ and rotor tilt motion damping $d = d(\Omega)$ to be wheel speed dependent.[10] Therefore, as long as both tilt controllers are working independently of each other, the overall satellite nutation in space will be unstable. Equation (22) reveals that increasing $c(\Omega)$ diminishes $\delta_s$, but does not change its sign, while a modification of $d(\Omega)$ has no effect.

A completely different dynamic behavior can be achieved if the tilt bearing control circuits are coupled in a proper way. This has been demonstrated at the DFVLR using the vernier gimballing facility of the MDR 100-1. In Eq. (7) internal interaction has been introduced into the equations of motion such as a feedback system, while air bearing testing of the RCPM 50 has proven that this method is adequate. The vector $q_w$ of passive internal interaction is now supplemented by an additional feedback of the tilt controllers due to

$$q_{w,e} = q_w + q_e, \quad q_e = E(D\dot{\Gamma}_w + C\Gamma_w) \tag{24}$$

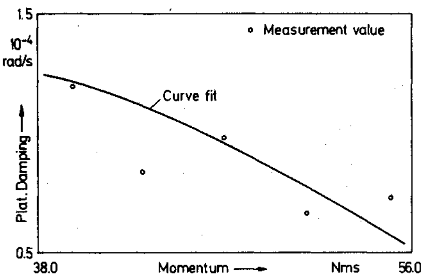


**Fig. 4 Decreasing platform damping ($-\delta_s$) with increasing wheel speed.**

**Table 1 Wheel parameters, RCPM 50**

| Parameter | Dimension |
|---|---|
| Spin axis m.o.i. | $I_w = 60.50 \cdot 10^{-3}$ kg$^2$ |
| Transverse m.o.i. | $I = 38.75 \cdot 10^{-3}$ kg$^2$ |
| Angular momentum | $H_w = 50$ Nms |
| Rotation speed | $\Omega = 826.5$ rad/s |
| Radial bearing stiffness | $c_w = 901.8$ Nm/rad |
| Coefficient of stator energy dissipation | $d_w = 0.2$ Nms/rad |
| Rotary damping | $k_w = 0.01$ Nms/rad |
| Damping from laboratory environment | $l_s = 0.025$ Nms/rad |

where

$$E=\begin{bmatrix} 0 & \epsilon \\ \\ -\epsilon & 0 \end{bmatrix} \tag{25}$$

is the skew symmetric matrix of interaxes cross coupling. The special form of $q_e$ is required by the controller hardware characteristics of the MDR 100-1. Inserting Eqs. (24) and (25) into Eqs. (9) and (10) and performing the approximate eigenvalue analysis as described before, Eq. (23) is modified to

$$\delta_s=\frac{(k_w\Omega+\epsilon c_w)H_w^3}{I_s^2(c_w+2H_w^2/I_s)^2}-\frac{l_s}{I_s} \tag{26}$$

while $\omega_s$ remains unchanged. Stability of the overall nutation now depends on the sign of $(k_w\Omega+\epsilon c_w)$. With $\epsilon=-0.2$, the test results are given in Fig. 5.

Figure 5 reveals that overall nutation of the air bearing platform is stable and, moreover, contrary to the RCPM 50, platform damping increases with increasing wheel speed. In either case, however, a good agreement between measurement data and analytical eigenvalue approach can be stated. A curve fit, according to Eq. (26), also supplemented by air bearing pendulum tests, demonstrates that damping is now provided by the cross coupling $\epsilon$. Table 2 identifies the following wheel data.

The actually stabilizing effect of bearing tilt controller interaxes cross coupling is also demonstrated by Fig. 6, where $|\epsilon|=0.2$ has been varied from $-10\%$ up to $+30\%$ with constant angular momentum. It confirms that $\delta_s$ increases nearly linearly with increasing $|\epsilon|$ according to Eq. (26). With $\epsilon\approx-0.18$, platform nutation is changed from a stable to an unstable motion.

In accordance with Eq. (26), design goals of active magnetic bearings are contrary to the passive bearing design. While rotary damping also must be kept small, a good damping of the overall nutation implies a proper amount of interaxes cross coupling, together with a small radial bearing stiffness. To minimize the power consumption of the active bearings, an optimal choice of $c_w$, $d_w$, $\epsilon$ is possible, which is now under investigation at the DFVLR.

## Summary

A linearized mathematical model of a symmetric despun satellite equipped with a magnetic bearing momentum wheel has been presented. Due to simplifications according to the physics of the system, an analytical eigenvalue approach is possible which permits a quantitative assessment of bearing design impact on the overall nutation. This approximate analysis has been verified by air bearing testing of two actual wheel configurations.

**Table 2 Wheel parameters, MDR 100-1**

| Parameter | Dimension |
|---|---|
| Spin axis m.o.i. | $I_w=59.2\cdot10^{-3}\,\text{kg}^2$ |
| Transverse m.o.i. | $I=33.7\cdot10^{-3}\,\text{kg}^2$ |
| Angular momentum (nom.) | $H_{w,\,\text{nom}}=100$ Nms |
| Angular momentum | $H_w=51$ Nms |
| Rotation speed | $\Omega=861.5$ rad/s |
| Radial bearing stiffness (51 Nms) | $c_w=900$ Nm/rad |
| Coefficient of tilt motion damping (51 Nms) | $d_w=31.7$ Nms/rad |
| Rotary damping | $k_w=0.06$ Nms/rad |
| Cross coupling | $\epsilon=-0.2$ |
| Damping from laboratory environment | $l_s=0.023$ Nms/rad |

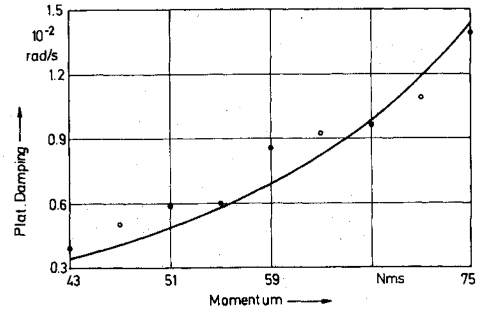


**Fig. 5 Increasing platform damping ($-\delta_s$) with increasing wheel speed.**

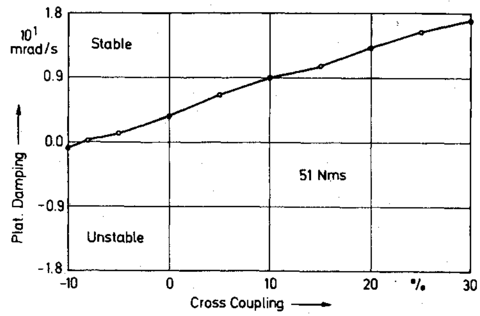


**Fig. 6 Increasing platform damping ($-\delta_s$) with $\epsilon$-variation from $-10\%$ up to 30%.**

The analysis reveals that with passive magnetic bearings, overall nutational instability is caused by rotor energy dissipation. The real part of the corresponding eigenvalue is dependent only on the overall transverse moment of inertia. Being proportional to the fourth power of the rotation speed, it indicates that the problem of passive open loop stability is most significant for satellites, where high precision pointing accuracy requires a large angular momentum bias.

Considering active magnetic bearings, according to the approximative eigenvalue analysis the nutational instability can be avoided by interaxes cross coupling of the bearing tilt controllers. Thus, open loop stability of satellites can be achieved by a proper phase shift of the internal bearing reaction torques.

## Appendix

Deriving the equations of motions of a two-body angular momentum storage system, the Lagrange formalism in quasicoordinate formulation[11] yields the rotational equation of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\Omega_{1,2}}\right)+\tilde{\omega}_1\left(\frac{\partial T}{\partial\Omega_{1,2}}\right)+\left(\frac{d\rho_{1,2}}{dt}\right)^{\approx}\left(\frac{\partial T}{\partial v_{1,2}}\right)=Q_{1,2} \tag{A1}$$

for Bodies 1 and 2, respectively. With $\omega_1$ as the overall angular velocity with respect to the inertial frame, Body 1 serves as a reference of the vector components. Assuming that the relative translation of Body 2 with respect to Body 1 (here the small relative translation of the rotor relative to the satellite fixed bearing elements) does not change the overall center of mass and the moments of inertia significantly, then the product on the left side, compromising the distance from the inertial reference $\rho$ and the linear velocity $v$, is approximately zero. Rotation and translation are decoupled and the investigation can be concentrated on the relative rotations $\Omega_{1,2}$.

Applying formalism (A1) to Bodies 1 and 2 separately and combining the resulting equations by elimination of the interaction torques yield the following general nonlinear matrix-vector equations[10]

$$I_1\dot{\omega}_1+\tilde{\omega}_1 I_1\omega_1+{}^1[I_2\dot{\Omega}_2]+\tilde{\omega}_1{}^1[I_2\Omega_2]=Q_1$$

$$^1I_2\dot{\omega}_1+\tilde{\omega}_1{}^1I_2\omega_1+{}^1[I_2\dot{\Omega}_2]+\tilde{\omega}_1{}^1[I_2\Omega_2]={}^1Q_2-{}^1q_2 \quad \text{(A2)}$$

where the upper left index[1] denotes that all vector components must be transformed to the reference frame of Body 1. The linearization procedure now depends mainly on the transformation.

Assuming that the transformation is described by Matrix S comprising sine- and cosine-functions of the vector of attitude angles $\mathbf{\Gamma}_2$, then a linearized representation of $S$ is given by

$$S\approx U+\tilde{\Gamma}_2 \quad \text{(A3)}$$

where $U$ is the unit matrix. In Eq. (A3) it has been assumed that $S$ is no function of the fast varying angle of the rotor (Body 2) about the nominal spin axis, which implies Body 2 to be symmetric. Performing the transformation yields

$$^1I_2=I_2+\tilde{\Gamma}_2 I_2+I_2\tilde{\Gamma}_2{}^T$$

$$^1\Omega_2=\dot{\Gamma}_2+\tilde{\Gamma}_2\Omega_2+\Omega_2 \quad \text{(A4)}$$

where $\Omega_2$ now only comprises the nonlinear part of the angular velocity. In the same way, it results for $\omega_1$

$$\omega_1=\dot{\Gamma}_1+(U+\tilde{\Gamma}_1)^T\omega_0 \quad \text{(A5)}$$

where $\omega_0$ is the angular velocity of a reference frame, to which the linearized rotation of Body 1 is related.

Inserting Eqs. (A4) and (A5) into Eq. (A2), while taking into account the general relationship $\tilde{a}b=-\tilde{b}a$ and denoting $I_2\Omega_2=H_2$, results in a set of linearized equations of motion, which is given by Eqs. (1) and (2).

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